CODE - A

TEST ID 002015

NTPL

ALL INDIA TEST SERIES

JEE (Main) - 2020

Batch - 2001+2002+2003+2005

PART TEST - 5

Time : 3 Hours | ***Maximum Marks : 300***

Syllabus Covered

Physics : Work, Power & Energy, Center of Mass, Conservation of Momentum, Magnetics, EMI.

Chemistry : P- Block Elements, D and F Block, Environmental Chemistry, Acid, Amines, Biomolecules, Atomic structure.

Mathematics : Trigonometric ratios and identities, Trigonometric equations and their solutions, Inverse trigonometric function, Solution of triangles.

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **75 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Mathematics).
10. **Section I** contains **20 questions**. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.
11. **Section II** contains **5 questions**. The answer to each question is a ***NUMERICAL VALUE.*** Each question carries **+4 marks** for correct answer. In all other cases **zero (0)** mark will be awarded for incorrect answer in this section.

Name of the Candidate (in Capitals) ______________________

Test Centre ______________________ Centre Code ______________________

Candidate's Signature ______________________ Invigilator's Signature ______________________

PART - I PHYSICS

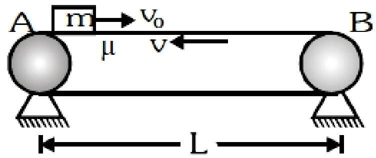
SECTION 1 (Maximum Marks: 80)

This section contains **TWENTY (20)** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. A block of mass m moving with speed v compresses a spring through distance x before its speed is halved. What is the value of spring constant?

(a) $\frac{3mv^2}{4x^2}$ (b) $\frac{mv^2}{4x^2}$ (c) $\frac{mv^2}{2x^2}$ (d) $\frac{2mv^2}{x^2}$

2. With what minimum velocity v_0 should block be projected from left end A towards end B such that it reaches the other end B of conveyer belt moving with constant velocity v? Friction coefficient between block and belt is μ.



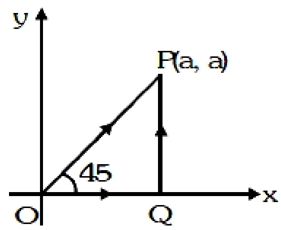
(a) $\sqrt{\mu g L}$ (b) $\sqrt{2\mu g L}$

(c) $\sqrt{3\mu g L}$ (c) $2\sqrt{\mu g L}$

3. A light spring of length 20 cm and force constant 2 N/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The height h from the surface of the table at which the block will have the maximum velocity is:

(a) 20 cm (b) 15 cm (c) 10 cm (d) 5cm

4. A particle is moved from (0, 0) to (a, a) under a force $\vec{F} = \left(3\tilde{i} + 4\tilde{j}\right)$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then:



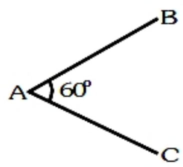
(a) $W_1 = W_2$

(b) $W_1 = 2W_2$

(c) $W_2 = 2W_1$

(d) $W_2 = 4W_1$

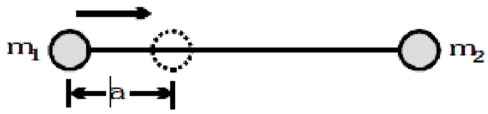
5. A uniform wire of length ℓ is bent into the shape of 'V' as shown. The distance of its centre of mass from the vertex A is



(a) $\ell / 2$ (b) $\frac{\ell\sqrt{3}}{4}$

(c) $\frac{\ell\sqrt{3}}{8}$ (d) None of these

6. Considering a system having two masses m_1 and m_2 in which first mass is pushed towards centre of mass by a distance a, the distance required to be moved for second mass to keep centre of mass at same position is



(a) $\frac{m_1}{m_2}a$ (b) $\frac{m_1 m_2}{a}$ (c) $\frac{m_2}{m_1}a$ (d) $\left(\frac{m_2 m_1}{m_1+m_2}\right)a$

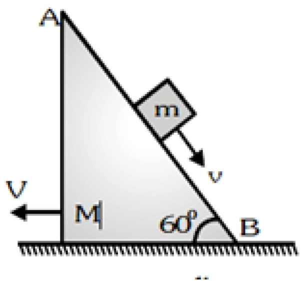
7. A particle of mass m is made to move with uniform speed v_0 along the perimeter of a regular hexagon, inscribed in a circle of radius R. The magnitude of impulse applied at each corner of the hexagon is

(a) $2mv_0 \sin\pi/6$ (b) $mv_0\sin\pi/6$ (c) $mv_0\sin\pi/3$ (d) $2mv_0 \sin\pi/3$

8. A cannon of mass 5m (including a shell of mass m) is at rest on a smooth horizontal ground, fires the shell with its barrel at an angle θ with the horizontal at a velocity u relative to cannon. Find the horizontal distance of the point where shell strikes the ground from the initial position of the cannon:

(a) $\frac{4u^2\sin 2\theta}{5g}$ (b) $\frac{u^2\sin 2\theta}{5g}$ (c) $\frac{3u^2\sin 2\theta}{5g}$ (d) $\frac{8u^2\sin 2\theta}{5g}$

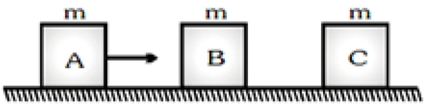
9. A particle of mass m = 0.1 kg is released from rest from a point A of a wedge of mass M = 2.4 kg free to slide on a frictionless horizontal plane. The particle slides down the smooth face AB of the wedge. When the velocity of the wedge is 0.2 m/s the velocity of the particle in m/s relative to the wedge is



(a) 4.8 (b) 5

(c) 7.5 (d) 10

10. Three blocks A, B and C each of mass m are placed on a surface as shown in the figure. Blocks B and C are initially at rest. Block A is moving to the right with speed v. It collides with block B and sticks to it. The A–B combination collides elastically with block C. Which of the following statement is (are) true about the velocity, of block B and C.



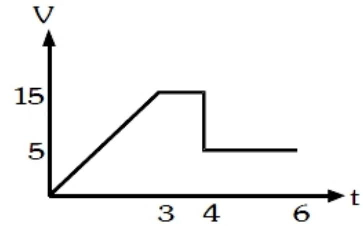
(a) Velocity of the block C after collision is 2/3 v towards right

(b) Velocity of the A–B combination after collision is $\frac{v}{3}$ towards left

(c) Velocity of the A–B combination after collision is $\frac{2}{3}v$ towards left

(d) Velocity of the block C after collision is $\frac{v}{3}$ towards right

11. The fig. shows the velocity as a function of the time for an object with mass 10 kg being pushed along a frictionless surface by external force. At t = 3s, the force stops pushing and the object moves freely. It then collides head on and sticks to another object of mass 25 kg, which of the following option is incorrect?



(a) External force acting on the system is 50 N

(b) Velocity of the 2nd particle just before the collision is 1 m/s

(c) Before collision both bodies are moving in the same direction

(d) Before collision, bodies are moving in opposite direction with respect to each other

12. An open water tight railway wagon of mass 5 x 10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

(a) 0.5 m/s (b) 2m/s (c) 1 m/s (d) 1.5 m/s

13. If the intensity of magnetic field at a point on the axis of current coil is half of that at the centre of the coil, then the distance of that point from the centre of the coil will be

(a) $\frac{R}{2}$ (b) R (c) $\frac{3R}{2}$ (d) $0.766R$

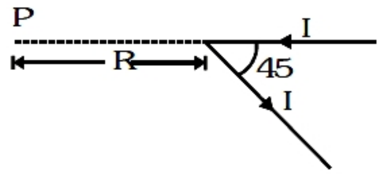
14. A helium nucleus is moving in a circular path of radius 0.8m. If it takes 2 sec to complete one revolution. Find out magnetic field produced at the centre of the circle.

(a) $\mu_0 10^{-19} T$ (b) $\frac{10^{-19}}{\mu_0}T$ (c) $2\times10^{-19}T$ (d) $\frac{2\times10^{-19}}{\mu_0}T$

15. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is:

(a) $(R_1/R_2)^{1/2}$ (b) R_2/R_1 (c) $(R_1/R_2)^2$ (d) R_1/R_2

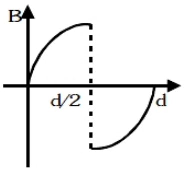
16. A long straight wire, carrying current I, is bent at its midpoint to form an angle of 45. Magnetic field at point P, distance R from point of bending is equal to

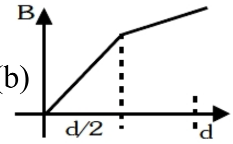


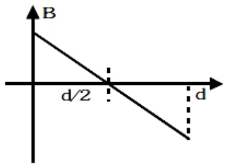
(a) $\frac{(\sqrt{2}-1)\mu_0 I}{4\pi R}$ (b) $\frac{\sqrt{2}+1\mu_0 I}{4\pi R}$

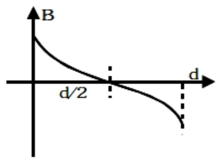
(c) $\frac{(\sqrt{2}+1)\mu_0 I}{4\sqrt{2}\pi R}$ (d) $\frac{(\sqrt{2}-1)\mu_0 I}{4\sqrt{2}\pi R}$

17. A uniform beam of positively charged particles is moving with a constant velocity parallel to another beam of negatively charged particles moving with the same velocity in opposite direction separated by a distance d. The variation of magnetic field B along a perpendicular line draw between the two beams is best represented by

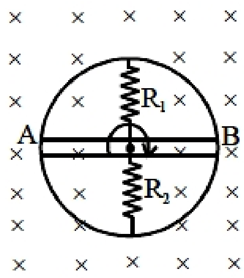
(a) 

(b) 

(c) 

(d) 

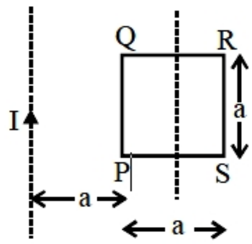
18. AB is resistance less conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown. The resistors R_1 and R_2 do not rotate. Then current through the resistor R_1 is–



(a) $\frac{B\omega r^2}{2R_1}$ (b) $\frac{B\omega r^2}{2R_2}$

(c) $\frac{B\omega r^2}{2R_1R_2}(R_1+R_2)$ (d) $\frac{B\omega r^2}{R(R_1+R_2)}$

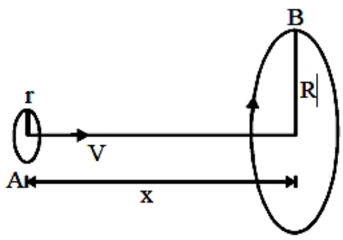
19. In the figure shown a square loop PQRS of side 'a' and resistance 'r' is placed in near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid point of the side QR and PS. The total amount of charge which passes through any point of the loop during rotation is



(a) $\frac{\mu_0 Ia}{2\pi r}\ell n2$ (b) $\frac{\mu_0 Ia}{\pi r}\ell n2$ (c) $\frac{\mu_0 Ia^2}{2\pi r}$

(d) Cannot be found because time of rotation not give

20. Loop A of radius (r << R) moves towards loop B with a constant velocity V in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced emf in loop A is maximum



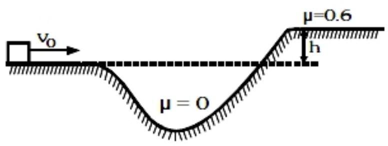
(a) R (b) $\frac{R}{\sqrt{2}}$

(c) $\frac{R}{2}$ (d) $R\left(1-\frac{1}{\sqrt{2}}\right)$

SECTION 2 (Maximum Marks: 20)

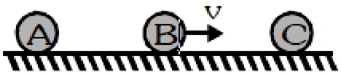
This section contains **FIVE (05)** questions. The answer to each question is a **NUMERICAL VALUE.** Each question carries **+4 marks** for correct answer. In all other cases **zero (0)** mark will be awarded for incorrect answer in this section.

21. In the figure, a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless untill the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6 m/s, the height difference h is 1.1 m and the coefficient of kinetic friction μ is 0.7. The value of d is in (m) ($g = 10 m/s^2$)

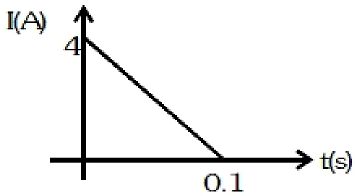


22. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then the position of centre of mass at t =1 s is in m

23. Three balls A, B and C ($m_A = m_C = 4m_B$) are placed on a smooth horizontal surface. Ball B collides with ball C with an initial velocity v as shown in the figure. Total number of collisions between the balls will be (All collisions are elastic)



24. When magnetic flux through a coil is changed, the variation of induced current in the coil with time is as shown in graph. If resistance of coil is $10\,\Omega$, then the total change in flux of coil will be in SI unit



25. For a inductor coil L = 0.04 H then work done by source to establish a current of 5A in it is m joule

PART - II CHEMISTRY

SECTION 1 (Maximum Marks: 80)

This section contains **TWENTY (20)** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

26. The element that forms compounds in nine different oxidation states is:

(a) N (b) P (c) As (d) Bi

27. Under normal conditions, which of the following hydride is non-combustible?

(a) NH_3 (b) PH_3 (c) AsH_3 (d) SbH_3

28. The compound molecular in nature in gas phase but ionic in solid state is

(a) PCl_5 (b) CCl_4 (c) PCl_3 (d) $POCl_3$

29. Which one of the following properties is not shown by NO?

(a) It is neutral oxide

(b) It combines with oxygen to form nitrogen dioxide

(c) Its bond order is 2.5

(d) It is diamagnetic in gaseous state

30. Which of the following is not oxidised by O_3 gas?

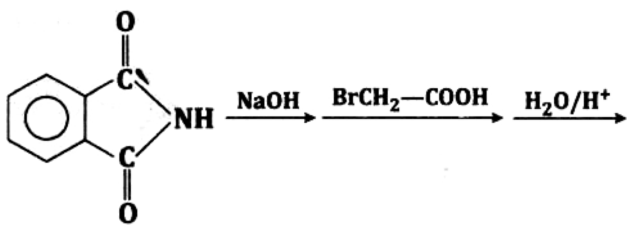
(a) KI (b) $FeSO_4$ (c) $KMnO_4$ (d) K_2MnO_4

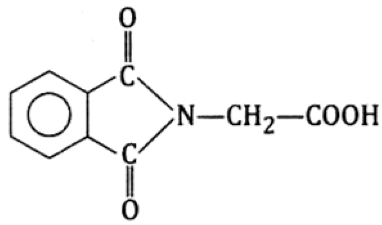
31. The correct order of solubility in water for He, Ne, Ar, Kr, Xe is:

(a) $He > Ne > Ar > Kr > Xe$ (b) $Ne > Ar > Kr > He > Xe$

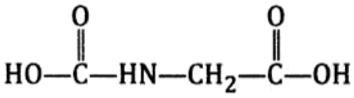
(c) $Xe > Kr > Ar > Ne > He$ (d) $Ar > Ne > He > Kr > Xe$

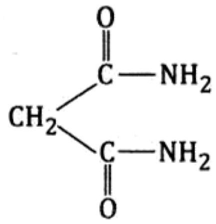
32. What is the final product in the following reaction?

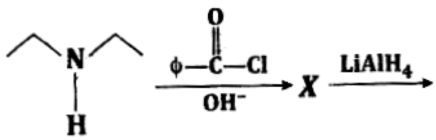


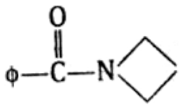
(a) 

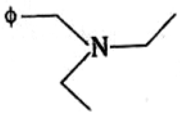
(b) $H_3N — CH_2 — COOH$

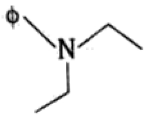
(c) 

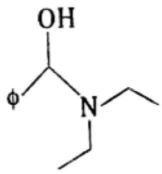
(d) 

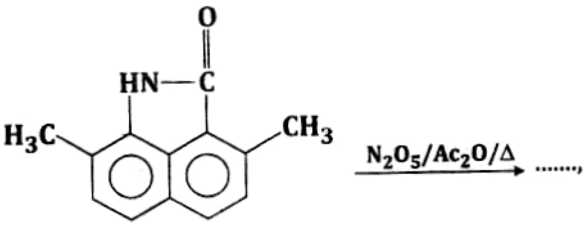
33.  the product 'Y' is:

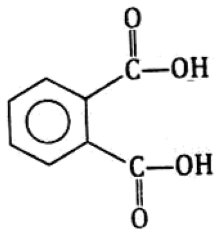
(a) 

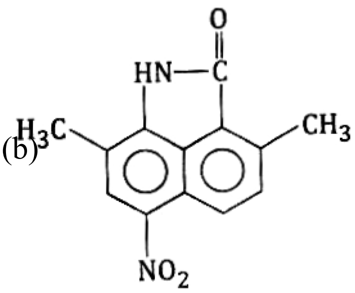
(b) 

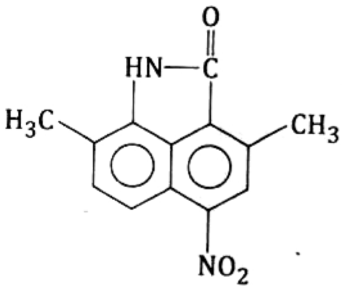
(c) 

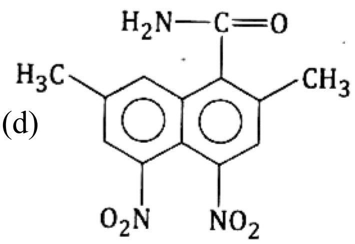
(d) 

34.  the major product is

(a) 

(b) 

(c) 

(d) 

35. Consider the reaction sequence and decide the compound ‘A’

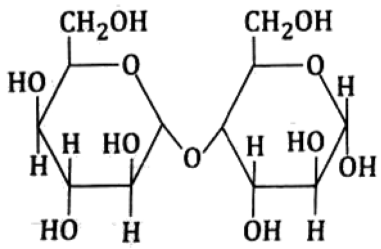
$$(A)\xrightarrow{\text{Reduction}}(B)\xrightarrow{HNO_2}CH_3-CH_2-OH$$

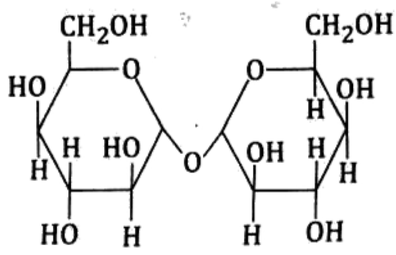
(a) CH_3-CH_2N (b) CH_3-CN (c) CH_3NC (d) CH_3CH_2-CN

36. Which of the following biomolecules is insoluble in water?

(a) $\alpha-$Keratin (b) Hemoglobin (c) Ribonuclease (d) Adenine

37. Which of the following will reduced Tollens’ reagent?

(I) 

(II) 

(a) I (b) II (c) Both I and II (d) None of these

38. Complete hydrolysis of cellulose gives:

(a) L-glucose (b) D-fructose (d) D-ribose (d) D-glucose

39. RNA is different from DNA, because RNA contains:

(a) ribose sugar and thymine (b) ribose sugar and uracil

(c) deoxyribose sugar and uracil (d) deoxyribose sugar and thymine

40. Which amino acid is optically inactive?

(a) Glycine (b) Serine (c) Alanine (d) All of these

41. Penicillin is:

(a) analgesic (b) antipyretic (c) antimalarial (d) antibiotic

42. Which of the following is an anionic detergent?

(a) Cetyltrimethyl ammonium bromide (b) Glyceryl oleate

(c) Sodium stearate (d) Sodium lauryl sulphate

43. Which of the following is non-addiction analgesic?

(a) Heroin (b) Morphine (c) Diazepam (d) Paracetamol

44. Chemically, alkaloid heroin is diacetate of:

(a) benzene (b) ethyne (c) morphine (d) aspirin

45. The first discovered antibiotic is

(a) Streptomycin (b) Penicillin (c) Chloramphenicol (d) Amoxycillin

SECTION 2 (Maximum Marks: 20)

This section contains **FIVE (05)** questions. The answer to each question is a **NUMERICAL VALUE.** Each question carries **+4 marks** for correct answer. In all other cases **zero (0)** mark will be awarded for incorrect answer in this section.

46. Visible spectrum contains light of following colours "violet-indigo-Blue-green-yellow-orange-red" (VIBGYOR). It's frequency ranges from violet (7.5×10^{14} Hz) to red (0.4×10^{14} Hz). Find out the maximum wavelength in this range.

47. A certain particle carries $2.\ 5 \times 10^{-16}$C of static electric charge. Calculate the number of electrons present in it.

48. In Milikan's experiment, static electric charge on the oil drops has been obtained by shining X-rays. If the static electric charge on the oil drop is $-\ 1.282 \times 10^{-18}$ C. Calculate the number of electrons present on it.

49. Find out the number of waves made by a Bohr electron in one complete revolution in it's 3^{rd} orbit.

50. H-atom is excited by a photon and this excited atom comes back to the ground state by loss of two photons of wavelength 4341.7 Å and 1215.6 Å. Find out principal quantum number of electron in excited state of H-atom –

PART - III MATHEMATICS

SECTION 1 (Maximum Marks: 80)

This section contains **TWENTY (20)** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

51. The sum $\frac{1}{\sin 45^\circ \sin 46^\circ}+\frac{1}{\sin 47^\circ \sin 48^\circ}+\frac{1}{\sin 49^\circ \sin 50^\circ}+...+\frac{1}{\sin 133^\circ \sin 134^\circ}$ is equal to

(a) $\sec(1^\circ)$ (b) $\operatorname{cosec}(1^\circ)$ (c) $\cot(1^\circ)$ (d) None of these

52. Set of values of x lying in $[0, 2\pi]$ satisfying the inequality $|\sin x| > 2\sin^2 x$ contains

(a) $\left(0, \frac{\pi}{6}\right)\cup\left(\pi, \frac{7\pi}{6}\right)$ (b) $\left(0, \frac{7\pi}{6}\right)$ (c) $\frac{\pi}{6}$ (d) None of these

53. If x_1 and x_2 are two distinct roots of the equation $a\cos x + b\sin x = c$, then $\tan\frac{x_1+x_2}{2}$ is equal to

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $\frac{a}{c}$

54. The minimum value of the function $f(x)=\frac{\sin x}{\sqrt{1-\cos^2 x}}+\frac{\cos x}{\sqrt{1-\sin^2 x}}+\frac{\tan x}{\sqrt{\sec^2 x-1}}+\frac{\cot x}{\sqrt{\operatorname{cosec}^2 x-1}}$ whenever it is defined is

(a) 4 (b) –2 (c) 0 (d) 2

55. In which one of the following intervals the inequality $\sin x < \cos x < \tan x < \cot x$ can hold good?

(a) $\left(\frac{7\pi}{4}, 2\pi\right)$ (b) $\left(\frac{3\pi}{4}, \pi\right)$ (c) $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$ (d) None of these

56. The general solution of $8\tan^2\frac{x}{2} = 1+\sec x$ is

(a) $x = 2n\pi \pm \cos^{-1}\left(\frac{-1}{3}\right)$ (b) $x = 2n\pi \pm \frac{\pi}{6}$ (c) $x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$ (d) None of these

57. The general solution of $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \cdot \tan 4\theta \cdot \tan 7\theta$

(a) $\theta = \frac{n\pi}{4}$ (b) $\theta = \frac{n\pi}{12}$ (c) $\theta = \frac{n\pi}{2}$ (d) None of these

58. The solution of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is

(a) $x = 0$
(b) $x = \sin^{-1}\left[\log\left(2-\sqrt{5}\right)\right]$
(c) no real solution
(d) None of the above

59. If in a ΔABC, $b = \sqrt{3}$, $c = 1$ and $B - C = 90°$, then $\angle A$ is

(a) 30° (b) 45° (c) 75° (d) 15°

60. In any ΔABC, if $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in AP, then a, b, c are in

(a) AP (b) GP (c) HP (d) None of these

61. A triangle has vertices A, B and C and the respectively opposite sides have lengths a, b and c. This triangle is inscribed in a circle of radius R. If $b = c = 1$ and the altitude from A to side BC has length $\sqrt{\frac{2}{3}}$, then R equals

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{2\sqrt{2}}$

62. Consider a ΔABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C, respectively. If $a = 1$, $b = 3$ and $C = 60°$, then $\sin^2 B$ is equal to

(a) $\frac{27}{28}$ (b) $\frac{3}{28}$ (c) $\frac{81}{28}$ (d) $\frac{1}{3}$

63. The angle of elevation of tower from a point A due south of it is 30° and from a point B due west of it is 45°. If the height of the tower be 100 m, then $AB =$

(a) 150 m (b) 200 m (c) 173.2 m (d) 141.4 m

64. The value of $5 \cdot \cot\left(\sum_{k=1}^{5} \cot^{-1}\left(k^2 + k + 1\right)\right)$ is equal to

(a) $\frac{5}{2}$ (b) 7 (c) –7 (d) $\frac{7}{2}$

65. Range of $f(x) = \sin^{-1}\log[x] + \log\left(\sin^{-1}[x]\right)$, where [ ] denotes GIF is

(a) {1} (b) {0} (c) $\left\{\log\frac{\pi}{2}\right\}$ (d) None of these

66. If the equation $5\operatorname{arc}\tan\left(x^2+x+k\right)+3\ \operatorname{arc}\cot\left(x^2+x+k\right)=2\pi$, has two distinct solutions, then the range of k, is

(a) $\left(0, \frac{5}{4}\right]$ (b) $\left(-\infty, \frac{5}{4}\right)$ (c) $\left(\frac{5}{4}, \infty\right)$ (d) $\left(-\infty, \frac{5}{4}\right]$

67. If $\cot^{-1}\left(\frac{n^2-10n+21\cdot 6}{\pi}\right)>\frac{\pi}{6}$, $n\in N$, then find the minimum value of n.

(a) 2 (b) 3 (c) 4 (d) None of these

68. Find the set of values of k for which $x^2-kx+\sin^{-1}\left(\sin 4\right)>0$ for all real x.

(a) R (b) $\left(-\frac{\pi}{4}, \pi\right)$ (c) ϕ (d) None of these

69. Solve for $x:\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$

(a) -1 (b) 1 (c) 2 (d) None of these

70. Find the sum of greatest and least value of; $\left(\sin^{-1}x\right)^3+\left(\cos^{-1}x\right)^3$.

(a) $\frac{6\pi^3}{8}$ (b) $\frac{7\pi^3}{8}$ (c) $\frac{9\pi^3}{8}$ (d) None of these

SECTION 2 (Maximum Marks: 20)

This section contains **FIVE (05)** questions. The answer to each question is a **NUMERICAL VALUE.** Each question carries **+4 marks** for correct answer. In all other cases **zero (0)** mark will be awarded for incorrect answer in this section.

71. The number of ordered pairs (x, y), when $x, y\in[0, 10]$ satisfying $\left(\sqrt{\sin^2 x-\sin x+\frac{1}{2}}\right)\cdot 2^{\sec^2 y}\le 1$ is

72. If $\cos 5\theta=a\cos\theta+b\cos^3\theta+c\cos^5\theta+d$, then find the value of $a+b+c+d$

73. If $a^2-2a\cos x+1=674$ and $\tan\left(\frac{x}{2}\right)=7$ then the integral value of a is

74. The number of solutions of the equation $\sin\left(\frac{\pi x}{2\sqrt{3}}\right)=x^2-2\sqrt{3}x+4$ is

75. Total number of solutions of $\sin x=\frac{|x|}{10}$ is equal to

SOLUTION OF AITS JEE(MAIN) PART TEST – 5

PHYSICS									
1	2	3	4	5	6	7	8	9	10
A	**B**	**B**	**A**	**C**	**A**	**A**	**A**	**D**	**A**
11	12	13	14	15	16	17	18	19	20
D	**C**	**D**	**A**	**C**	**A**	**D**	**A**	**B**	**C**
21	22	23	24	25					
1	**6**	**2**	**2**	**1**					

CHEMISTRY									
26	27	28	29	30	31	32	33	34	35
A	**A**	**A**	**D**	**C**	**C**	**B**	**B**	**B**	**B**
36	37	38	39	40	41	42	43	44	45
A	**A**	**C**	**C**	**A**	**D**	**D**	**D**	**C**	**B**
46	47	48	49	50					
750	**1563**	**8**	**3**	**5**					

MATHS

51. **(b)**

$$T_1 = \frac{1}{\sin 1^\circ}\left[\frac{\sin(46^\circ - 45^\circ)}{\sin 45^\circ \sin 46^\circ}\right] = \frac{1}{\sin 1^\circ}[\cot 45^\circ - \cot 46^\circ]$$

$$T_2 = \frac{1}{\sin 1^\circ}\left[\frac{\sin(48^\circ - 47^\circ)}{\sin 48^\circ \sin 47^\circ}\right] = \frac{1}{\sin 1^\circ}[\cot 47^\circ - \cot 48^\circ]$$

$$T_l = \frac{1}{\sin 1^\circ}\left[\frac{\sin(133^\circ - 134^\circ)}{\sin 133^\circ \sin 134^\circ}\right] = \frac{1}{\sin 1^\circ}[\cot 133^\circ - \cot 134^\circ]$$

On adding

$$\sum_{r=1}^{l} T_r = \frac{1}{\sin 1^\circ}\left[\{\cot 45^\circ + \cot 47^\circ + \cot 49^\circ + \ldots + \cot 133^\circ\} - \{\cot 46^\circ + \cot 48^\circ + \cot 50^\circ + \ldots + \cot 134^\circ\}\right]$$

$= \operatorname{cosec} 1^\circ$ [all terms cancelled except cot 45° remains]

52. **(a)**

$$|\sin x| > 2\sin^2 x$$

$$\Rightarrow |\sin x|(2|\sin x| - 1) < 0 \Rightarrow 0 < |\sin x| < \frac{1}{2} \Rightarrow x \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right) \cup \left(\pi, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$$

53. **(b)**

$$a\cos x + b\sin x = c$$

$$\Rightarrow \frac{a\left(1 - \tan^2 \frac{x}{2}\right)}{1 + \tan^2 \frac{x}{2}} + \frac{2b\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = c \Rightarrow (c + a)\tan^2 \frac{x}{2} - 2b + \tan \frac{x}{2} + c - a = 0$$

$\Rightarrow \tan\frac{x_1}{2}+\tan\frac{x_2}{2}=\frac{2b}{c+a},\ \tan\frac{x_1}{2}\cdot\tan\frac{x_2}{2}=\frac{c-a}{c+a}$

Thus, $\tan\left(\frac{x_1+x_2}{2}\right)=\frac{\tan\frac{x_1}{2}+\tan\frac{x_2}{2}}{1-\tan\frac{x_1}{2}\tan\frac{x_2}{2}}=\frac{\frac{2b}{c+a}}{1-\left(\frac{c-a}{c+a}\right)}=\frac{b^2}{a}$

54. **(b)**

$f(x)=\frac{\sin x}{\sqrt{1-\cos^2 x}}+\frac{\cos x}{\sqrt{1-\sin^2 x}}+\frac{\tan x}{\sqrt{\sec^2 x-1}}+\frac{\cot x}{\sqrt{\operatorname{cosec}^2 x-1}}$

$=\frac{\sin x}{|\sin x|}+\frac{\cos x}{|\cos x|}+\frac{\tan x}{|\tan x|}+\frac{\cot x}{|\cot x|}=\begin{cases}4, & x\in \text{ 1st quadrant}\\ -2, & x\in \text{ 2nd quadrant}\\ 0, & x\in \text{ 3rd quadrant}\\ -2, & x\in \text{ 4th quadrant}\end{cases}$

$f(x)_{\min}=-2$

55. **(d)**

In the second quadrant, $\sin x<\cos x$ is false, as $\sin x$ is positive and $\cos x$ is negative.

In the fourth quadrant, $\cos x<\tan x$ is false, as $\cos x$ is positive and $\tan x$ is negative.

In the third quadrant, i.e. $\left(\frac{5\pi}{4},\frac{3\pi}{2}\right)$ if $\tan x<\cot x$ then $\tan^2 x<1$, which is false.

Now, $\sin x<\cos x$ is true in $\left(0,\frac{\pi}{4}\right)$ and $\tan x<\cot x$ is also true.

Further, $\cos x<\tan x$, as $\tan x=\frac{(\sin x)}{(\cos x)}$ and $\cos x<1$.

56. **(c)**

Consider, $\tan^2\frac{x}{2}=1+\sec x$

$\Rightarrow 8\left(\frac{1-\cos x}{1+\cos x}\right)=1+\frac{1}{\cos x}\ \Rightarrow 8\cos x-8\cos^2 x=(1+\cos x)^2\ \Rightarrow 8\cos x-8\cos^2 x=1+\cos^2 x+2\cos x$

$\Rightarrow\ 9\cos^2 x-6\cos x+1=0\ \Rightarrow (3\cos x-1)^2=0\ \Rightarrow \cos x=\frac{1}{3}=\cos\alpha$

$\Rightarrow x=2n\pi\pm\alpha$ where 'α' $=\cos^{-1}\frac{1}{3}$ [By using $\cos\theta=\cos\alpha\Rightarrow\theta=2n\pi\pm\alpha$]

$\Rightarrow x=2n\pi\pm\cos^{-1}\left(\frac{1}{3}\right)$

57. **(b)**

We have $\tan\theta+\tan 4\theta+\tan 7\theta=\tan\theta\cdot\tan 4\theta\cdot\tan 7\theta$

$\Rightarrow\ \tan\theta+\tan 4\theta=-\tan 7\theta+\tan\theta\cdot\tan 4\theta\cdot\tan 7\theta$

$\Rightarrow\ \tan\theta+\tan 4\theta=-\tan 7\theta\left(1-\tan\theta\cdot\tan 4\theta\right)$

$\Rightarrow\ \dfrac{\tan\theta+\tan 4\theta}{1-\tan\theta\cdot\tan 4\theta}=-\tan 7\theta\ \Rightarrow\ \tan(\theta+4\theta)=-\tan 7\theta\ \Rightarrow\ \tan 5\theta=\tan(-7\theta)$

$\Rightarrow 5\theta=n\pi+(-7\theta)\ \Rightarrow 12\theta=n\pi\ \Rightarrow \theta=\dfrac{n\pi}{12}$ (where $n\in I$)

58. **(c)**

Put $e^{\sin x}=t$

Given equation becomes $t^2-4t-1=0\ \Rightarrow\ t=2\pm\sqrt{5}\ \Rightarrow e^{\sin x}=2\pm\sqrt{5}$

Either $\sin x=\log_e\left(2+\sqrt{5}\right)$ or $\sin x=\log_e\left(2-\sqrt{5}\right)$

As $2+\sqrt{5}>e$ or as $\left(2-\sqrt{5}\right)$ is (–ve) and log is not defined for (–ve) values.

$\Rightarrow\ \sin x>1\Rightarrow$ no solution.

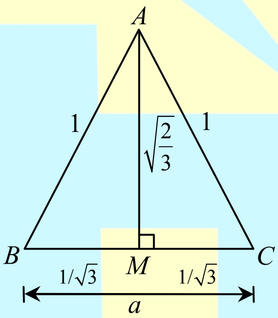
59. **(a)**

60. **(a)**

61. **(d)**

$a=\dfrac{2}{\sqrt{3}}$ and $\Delta=\dfrac{1}{2}ah=\dfrac{1}{2}\cdot\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{2}}{\sqrt{3}}=\dfrac{\sqrt{2}}{3}$

$\therefore\ R=\dfrac{abc}{4\Delta}=\dfrac{(2)(1)(3)}{\sqrt{3}(4)\sqrt{2}}=\dfrac{\sqrt{3}}{2\sqrt{2}}$

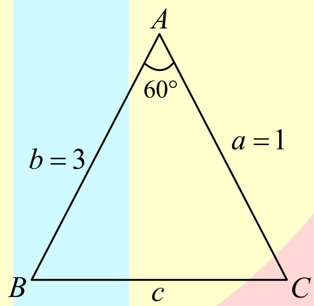


62. **(a)**

By Cosine law, $\cos 60^\circ=\dfrac{1^2+3^2-c^2}{2\cdot1\cdot3}\ \Rightarrow\ c=\sqrt{7}$

Now, $\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$ [By Sine law]

$\Rightarrow\ \sin B=\dfrac{b\sin C}{c}=\dfrac{3\times\frac{\sqrt{3}}{2}}{\sqrt{7}}=\dfrac{3\sqrt{3}}{2\sqrt{7}}$



Hence, $\sin^2 B=\dfrac{27}{28}$

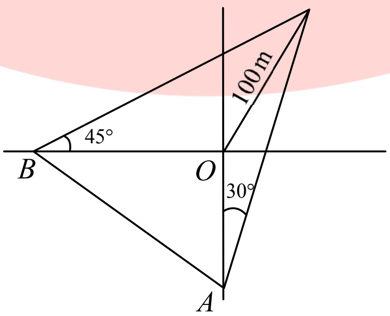
63. **(b)**

$OB=100\cot 45^\circ$

$OA=100\cot 30^\circ$

$AB=\sqrt{\left(OA^2+OB^2\right)}$

$AB=200\,\text{m}$



64. **(b)**

Consider

$$\sum_{k=1}^{5}\left(\tan^{-1}\left(\frac{(k+1)-k}{1+k(k+1)}\right)\right)=\sum_{k=1}^{5}\tan^{-1}(k+1)-\tan^{-1}k$$

Now, $T_1=\tan^{-1}(2)-\tan^{-1}(1)$;

$T_2=\tan^{-1}(3)-\tan^{-1}(2)$ and so on

Hence, $\sum_{k=1}^{5}\cot^{-1}(k^2+k+1)=\tan^{-1}(6)-\tan^{-1}(1)=\tan^{-1}\left(\frac{5}{7}\right)=\cot^{-1}\left(\frac{7}{5}\right)$

$\therefore\ 5\cot\left(\cot^{-1}\frac{7}{5}\right)=7$

65. **(c)**

Domain of $f(x)$ is $[1, 2)$

$\therefore$ Range is $\left\{\log\frac{\pi}{2}\right\}$

66. **(b)**

We have $2\pi=\frac{3\pi}{2}+2\tan^{-1}(x^2+x+k)$ $\quad\left(\text{As, } \tan^{-1}\alpha+\cot^{-1}\alpha=\frac{\pi}{2}\ \forall\,\alpha\in R\right)$

$\Rightarrow\ \tan^{-1}(x^2+x+k)=\frac{\pi}{4}\Rightarrow x^2+x+k=1\ \Rightarrow x^2+x+(k-1)=0$

$\therefore$ For required condition, put $D>0$

$\Rightarrow 1-4(k-1)>0\Rightarrow 5-4k>0\ \Rightarrow k<\frac{5}{4}$

67. **(b)**

We have, $\cot^{-1}\left(\frac{n^2-10n+21\cdot 6}{\pi}\right)>\frac{\pi}{6}$

$\Rightarrow\ \frac{n^2-10n+21\cdot 6}{\pi}<\cot\frac{\pi}{6}$ $\quad\{\text{as } \cot x \text{ is decreasing for } 0<x<\pi\}$

$\Rightarrow n^2-10n+21.6<\pi\sqrt{3}\ \Rightarrow n^2-10n+25+21.6-25<\pi\sqrt{3}\ \Rightarrow\ (n-5)^2<\pi\sqrt{3}+3\cdot 4$

$\Rightarrow -\sqrt{\sqrt{3}\pi+3\cdot 4}<n-5<\sqrt{\sqrt{3}\pi+3.4}$ $\quad\ldots$(i)

Since, $\sqrt{3}\pi=5.5$ nearly

$\therefore\ \sqrt{\sqrt{3}\pi+3\cdot 4}\sim\sqrt{8\cdot 9}\sim 2\cdot 9\ \Rightarrow\ 2\cdot 1<n<7\cdot 9$

$\therefore\ n=3,\ 4,\ 5,\ 6,\ 7$ $\quad\{\text{as } n\in N\}$

or minimum value of $n=3$

68. **(a)**

We know, $\sin^{-1}(\sin 4)=\sin^{-1}(\sin(\pi-4))=\pi-4$ $\quad\left\{\because -\frac{\pi}{2}<\pi-4<\frac{\pi}{2}\right\}$

$\therefore$ We have $x^2 - kx + \pi - 4 > 0$ for all $x \in R$

$\therefore$ $D < 0$, i.e. $k^2 - 4(\pi - 4) < 0$ or $k^2 + 4(4 - \pi) < 0$

which is not true for any real k. {as $k^2 + 4(4-\pi) > 0$ }

69. **(a)**

We have $\left(\tan^{-1} x\right)^2 + \left(\cot^{-1} x\right)^2 = \dfrac{5\pi^2}{8}$

$\Rightarrow \left(\tan^{-1} x + \cot^{-1} x\right)^2 - 2\tan^{-1} x.\cot^{-1} x = \dfrac{5\pi^2}{8}$

$\Rightarrow \left(\dfrac{\pi}{2}\right)^2 - 2\tan^{-1} x.\left(\dfrac{\pi}{2} - \tan^{-1} x\right) = \dfrac{5\pi^2}{8}$ $\left\{\because \tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2} \Rightarrow \cot^{-1} x = \dfrac{\pi}{2} - \tan^{-1} x\right\}$

$\Rightarrow \dfrac{\pi^2}{4} - 2\cdot\dfrac{\pi}{2}\cdot\tan^{-1} x + 2\left(\tan^{-1} x\right)^2 = \dfrac{5\pi^2}{8}$ $\Rightarrow 2\left(\tan^{-1} x\right)^2 - \pi\tan^{-1} x - \dfrac{3\pi^2}{8} = 0$

$\Rightarrow \tan^{-1} x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ $\Rightarrow \tan^{-1} x = -\dfrac{\pi}{4}$ $\Rightarrow x = -1$ $\left\{\text{neglecting } \tan^{-1}x = \dfrac{3\pi}{4} \text{ as } \tan^{-1}x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[\right\}$

70. **(d)**

We have, $\left(\sin^{-1} x\right)^3 + \left(\cos^{-1} x\right)^3 = \left(\sin^{-1} x + \cos^{-1} x\right)$ $\left[\left(\sin^{-1} x + \cos^{-1} x\right)^2 - 3\sin^{-1} x.\cos^{-1} x\right]$

$= \dfrac{\pi}{2}\left[\dfrac{\pi^2}{4} - 3\sin^{-1} x\left(\dfrac{\pi}{2} - \sin^{-1} x\right)\right]$ $= \dfrac{3\pi}{2}\left[\left(\sin^{-1} x\right)^2 - \dfrac{\pi}{2}\sin^{-1} x + \dfrac{\pi^2}{12}\right]$ $= \dfrac{3\pi}{2}\left[\left(\sin^{-1} x - \dfrac{\pi}{4}\right)^2 + \dfrac{\pi^2}{12} - \dfrac{\pi^2}{16}\right]$

$= \dfrac{3\pi}{2}\left[\left(\sin^{-1} - \dfrac{\pi}{4}\right)^2 + \dfrac{\pi^2}{48}\right]$

So, the least value is; $\dfrac{3\pi}{2}\cdot\dfrac{\pi^2}{48} = \dfrac{\pi^3}{32}$ $\left\{\text{when} \left(\sin^{-1} x - \dfrac{\pi}{4}\right) = 0\right\}$

Also, $\left(\sin^{-1} x - \dfrac{\pi}{4}\right)^2 \le \left(\dfrac{3\pi}{4}\right)^2$

$\therefore$ The greatest value $= \dfrac{3\pi}{2}\left[\dfrac{9\pi^2}{16} + \dfrac{\pi^2}{48}\right] = \dfrac{7\pi^3}{8}$

71. **(16)**

$\sqrt{\sin^2 x - \sin x + \dfrac{1}{2}} = \sqrt{\left(\sin x - \dfrac{1}{2}\right)^2 + \dfrac{1}{2}} \ge \dfrac{1}{2}$, $\forall x$ and $\sec^2 y \ge 1$, $\forall\, y$, so $2^{\sec^2 y} \ge 2$. Hence, the above inequality holds only for those values of x and y for which $\sin x = \dfrac{1}{2}$ and $\sec^2 y = 1$.

Hence, $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}$ and $y = 0, \pi, 2\pi, 3\pi$. Hence, required number of ordered pairs are 16.

72. **(1)**

Put $\theta = \frac{\pi}{2}$ in the given inequality, we get $d = 0$

Put $\theta = 0$ in the given inequality, we get

$$a + b + c + d = 1 \qquad \ldots(i)$$

So, (d) is correct and (c) is not correct.

Now differentiate both sides with respect to θ, we get

$$-5\sin\theta = -a\sin\theta - 3b\cos^2\theta\sin\theta - 5c\cos^4\theta\sin\theta \qquad \ldots(ii)$$

Put $\theta = \frac{\pi}{2}$, then $a = 5$

Again putting $\theta = \frac{\pi}{4}$ in the given expression or in (2), we get

$$4a + 2b + c = -4 \qquad \ldots(iv)$$

From (i), (iii) and (iv) we have $b = -20$ and $c = 16$

73. **(25)**

$$674 = a^2 - 2a\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)} + 1 = a^2 - 2a\times\frac{1-49}{1+49} + 1 = a^2 + 2a\times\frac{48}{50} + 1$$

$\Rightarrow 25a^2 + 48a - 673\times 25 = 0 \Rightarrow (a-25)(25a+673) = 0 \Rightarrow a = 25$ (taking the integral value of a)

74. **(1)**

We know that $-1 \le \sin\frac{\pi x}{2\sqrt{3}} \le 1$, therefore, we must have

$$-1 \le x^2 - 2\sqrt{3}x + 4 \le 1 \Rightarrow -1 \le \left(x-\sqrt{3}\right)^2 + 1 \le 1 \Rightarrow -2 \le \left(x-\sqrt{3}\right)^2 \le 0$$

But, square of a real number cannot be negative, therefore, we must have $\left(x-\sqrt{3}\right)^2 = 0$

$$\Rightarrow x = \sqrt{3}$$

Note that $x = \sqrt{3}$ satisfies the given equation.

75. **(6)**

Graphs of $y = \sin x$ and $y = \frac{|x|}{10}$ meet exactly six times. Hence, there are six solutions.